A SIMPLIFIED TWO-DIMENSIONAL BOUNDARY ELEMENT PROGRAM FOR ESTIMATING MULTIPLE-SEAM INTERACTIONS

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ABSTRACT

As a result of the fast depletion of unexploited or virgin coal fields, mine operators are forced to mine in unfavorable or multiple-seam conditions. The safe, productive exploitation of coal in multiple-seam conditions requires specific design technology. Presently, there are two general design approaches that the mining engineer can use to analyze seam interactions in multiple-seam mining, either simple empirical relationships or complex numerical models. Often, the empirical approaches are too general for the specific problem and the numerical models require too much time and expertise for the practicing engineer. This paper introduces a new computer program, LaM2D, designed specifically to enhance multiple seam mining operation by quickly and easily calculating, stress, displacement, subsidence, horizontal strain and safety factors associated with multiple seam mining conditions. The program is an implementation of a simplified, two-dimensional boundary-element method designed for modeling complex multiple-seam stress and displacement interactions. Most importantly, the program is greatly simplified by incorporating automatic overburden, interburden, coal and gob property generation thereby reducing typical modeling and solution time to a few minutes.

1. INTRODUCTION

A lot of information on pillar design, strata mechanics and support technology has been developed and implemented in the coal mining industry in order to minimize ground falls and better protect mine workers. This information has gone a long way in reducing the number of injuries and fatalities experienced in the mines. In the last decade, fatalities resulting from ground falls have
averaged 13 per year [1]. This is a substantial reduction, but mining is still classified as one of the most hazardous industries in the United States and groundfalls still account for 70% [1] of all fatalities in underground mines.

Due to the rare existence of virgin areas and the fast depletion of available reserves, mining company’s are faced with mining in more difficult underground conditions such as mining under (undermining) or above (overmining) pre-existing mines. From about 700 active underground mine in the United States in 2002 [2], it could be assumed that quite a few of them were operating in multiple-seam mining situation. In fact, studies estimate that 156 billion tons of coal, representing 68% of the United States minable coal reserves exists in multiple-seam conditions [3]. Review of literature indicates a large number of ground control problems are indeed often associated with multiple-seam situations [4, 5, 6]. In order to ensure safe productivity of mines existing in multiple-seam conditions, a clear understanding of seam interaction is very important. In the past, simple empirical computer programs were developed to estimate multiple-seam interactions [7, 8, 9], but these programs were found to be too general for specific problems. Presently, mining engineers are drawn to the use of numerical techniques for the estimation of multiple-seam interactions.

Numerical modeling as a tool for engineering design has been utilized for decades. During this time, the awareness of its relevance has greatly increased, and at the same time, considerable developments have been made at improving its accuracy. There are several general numerical techniques for solving engineering problems; one may choose to use finite-element, boundary-element, discrete-element, finite difference and/or a hybrid combination of the above to solve the problem at hand. Based on the mathematical approximations involved, numerical methods can be classified into two categories: domain methods and boundary methods. The domain methods are characterized by area and volume discretization of the problem domain. This method uses relatively large numbers of elements resulting in large systems of equation which usually consume large amounts of computer resources and require lengthy times for solution. The boundary element method (BEM), on the other hand, requires only the discretization of the boundaries of the domain of interest. Relative to subsurface mineral extraction, only the planar area of the seam is discretized in order to obtain solution for the desired problem. The boundary element method usually leads to smaller systems of equations, faster computing times and a reduced need for computing resources.

The Displacement Discontinuity (DD) method is one variation of the boundary element method that is usually the method of choice for the analysis of stresses and displacements in slit-like or thin seam openings. In this approach, the relative movement between the roof and floor of a mine is treated mathematically as a discontinuity in the displacement field of the surrounding media, in other words, a crack/slit in a continuum. The displacement discontinuity method has been used to create several well know computer packages for solving geomechanical problems in underground mines.

One of the first implementations of the displacement discontinuity method into a personal computer code for calculating stress and displacement from multiple seams was done by the U.S Bureau of Mines. The program was called MULSIM/BM. In 1992 [10, 11], MULSIM/BM was upgraded to MULSIM/NL by the addition of non-linear seam material models, calculations for determining the mechanical strain energy changes associated with the seam materials, and a capability for “stepping” though a series of mining sequences. A new displacement discontinuity program LaModel [12] was introduced in 1998, also by the U.S Bureau of Mines. LaModel implemented the same non-linear seam material models in MULSIM/NL but introduced a laminated overburden model. This model assumes the overburden to be a stack of horizontally lying strata which naturally increases the flexibility of the overburden. As a result, LaModel estimation of the stresses, displacements and surface subsidence are considered more accurate for sedimentary deposition as compared to the
homogeneous overburden model implemented in MULSIM/NL.

In this paper, a new two-dimensional boundary-element program (LaM2D) is introduced. This program implements the laminated overburden model [12, 13, 14] used in the 3D LaModel program, into a greatly simplified two-dimensional program. This 2D implementation of the laminated boundary-element program was specifically developed in order to provide a fast, simple, multiple-seam analysis program for the practicing engineer. LaM2D only requires that the user input the minimum geometric parameters from the multiple-seam mining situation. The program automatically incorporates default overburden, coal and gob properties to simplify the input, and inherently calculates pillar safety factors for the program output.

2. 2D LAMINATED OVERBURDEN MODEL

The LaM2D program is an implementation of Salamon’s [14] laminated overburden model into a two-dimensional displacement-discontinuity program. One of the first modern implementations of this model was for predicting subsidence due to longwall coal mines by Yang in 1992 [15]. More recently, this laminated model was implemented into the full-featured computer code, LaModel, by Heasley in 1998 [12]. The original derivation of the laminated overburden model is based on the theory of thin plates [16]. This derivation assumes that the overburden is a stack of horizontal lying strata laminations, each having the same elastic modulus, Poisson’s ratio and thickness (what has been termed as: “homogeneous laminations”). The interfaces between the strata laminations are free of shear stresses and cohesion, essentially frictionless. Heasley [12] in his PhD thesis developed the 3D LaModel program based on the fundamental equation for a laminated overburden with homogeneous stratification derived by Salamon [17]:

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} = \frac{2}{E \lambda} \sigma_i$$  \hspace{1cm} (1)

where:

- $s$ = the vertical seam convergence
- $\sigma_i$ = the vertical induced stress in the overburden
- $E$ = the elastic modulus of the overburden laminations
- $x$ = the horizontal coordinate
- $y$ = the lateral coordinate
- $\lambda$ = a property of the laminated overburden as defined by:

$$\lambda = \frac{t}{\sqrt{12(1-\nu^2)}}$$  \hspace{1cm} (2)

Equation 1 however, is a three dimensional (3D) numerical solution, where changes in the vertical stresses and displacement are monitored in the horizontal $x$ and $y$ coordinates. Hence, for a simplified solution in 2D, changes in the convergence in the out-of-plane ($y$-direction) are assumed to be zero (a plane strain analysis). In other words, the in-seam grid elements are assumed to be infinitely long in the out-of-plane ($y$) direction. As a result of the two dimensional plane strain assumption Equation 1 reduces to:

$$\frac{\partial^2 s}{\partial x^2} = \frac{2}{E \lambda} \sigma_i$$  \hspace{1cm} (3)

Equation 3 is then the fundamental two-dimensional (2D) laminated overburden model which relates the total induced stress, $\sigma_i$, at the seam level with the vertical displacement, $s$, in the seam for a 2D plane-strain solution as implemented in the new LaM2D program.

The total induced stress ($\sigma_i$) in the laminated overburden model is composed of different contributing stress factors. For example, in the simple case of just the overburden load acting on a completely open seam, the induced stress would be equal to the overburden stress ($\sigma_o$). However, when
there exists any seam material supporting the roof, the total induced stress will also include the support of the seam materials ($\sigma_e$). The amount of support provided by the seam material is typically a function of the seam convergence ($\sigma_e(s)$). For a linear elastic seam material the material induced stress would be a linear function of the seam convergence. However, with a strain-softening (yielding material) or strain-hardening (gob material), the material induced stress can easily assume a non-linear relationship to the seam convergence. Seam interaction as a result of the existence of a remote seam can also provide a component to the total induced stress, the multiple seam stress ($\sigma_m$). The last induced stress component considered is the surface effect stress ($\sigma_s$) due to a traction free plane at the ground surface. Therefore the total induced stress at the seam level is given by:

$$\sigma_r(s) = -\sigma_e + \sigma_r(s) - \sigma_e(s) - \sigma_s(s) \quad (4)$$

Since the induced stress is generally non-linear, the fundamental equation (Equation 3) must be solved numerically. In LaM2D, the fundamental equation is numerically solved using a central-difference approximation on an even grid with a Gauss-Seidel iteration using Successive Over-Relaxation (SOR).

$$\Delta s_l = \frac{O}{2} \left( s_{in} + (s_{in})^{\circ} - \Delta \sigma \frac{2}{AE} \right) \sigma_r(s) \quad (5)$$

Where $s$ is the convergence, $O$ is the over relaxation factor (with a value between 1 and 2), and $\Delta x$ is the grid dimension.

3. MULTIPLE-SEAM STRESS AND DISPLACEMENT INFLUENCE FUNCTION

The existence of remote seams above or below (Figure 2) a local seam causes seam interactions which affect both seams. The effect of seam convergence experienced in one seam results in both displacement and stress propagation to the other seam through the laminated overburden. The LaM2D program employs an influence-function methodology to propagate the displacement effects between the multiple seams. Essentially, an equation which relates the convergence in an element of the local seam to the displacements in the remote seam was utilized.

Using this influence-function, the multiple-seam displacement effect on the adjacent seam is determined by numerically integrating the finite incremental displacements from every element in the local seam to every element in the remote seam. Therefore, the total multiple-seam component of the displacement at any point in the remote seam is the sum of the many small influences from the elements in the local seam.

$$w(\Delta x, \Delta z) = \frac{s_l}{4\sqrt{\pi z|\Delta z|}} e^{-\frac{\Delta x^2}{4\Delta z}} \quad (6)$$

where:
- $w$ = the vertical displacement of the remote point
- $s_l$ = the convergence of the local element
- $\Delta x$ = the horizontal distance between the local and the remote elements
- $\Delta z$ = the vertical distance between the local and the remote elements
Equation 6 is the kernel equation for the two-dimensional influence function used for the solution of the remote displacement within the LaM2D program. To calculate the stress influence function, knowing that stress ($\sigma$) is related to changes in displacement by the elastic modulus ($E$)

$$\sigma = E \frac{dw}{dz}$$  

(7)

The two-dimensional influence function for remote vertical stress ($f_v$) in a laminated overburden due to local seam convergence can be determined by taking the derivative of Equation 6 and multiplying by the elastic modulus of the rock ($E$):

$$f_v = E \frac{dw}{dz} = s_i \frac{E}{8\sqrt{\pi z^3}} \left( \frac{x^2}{2z^2} - 1 \right) e^{-\frac{x^2}{4z}}$$  

(8)

This multiple-seam component of the vertical stress is combined with the virgin in-situ stress and the in-seam stress distribution (Equation 4) to get the “total induced stress” on each element of the seam.

4. LAM2D INPUT AND OUTPUT

Compared to most numerical modeling programs, the input and output for the LaM2D program was simplified as much as possible to make it very user-friendly. Typical numerical modeling programs involve parameter input, solution of the model based on the input parameters and an output or plot of the solution values. In many cases three separate programs are employed to perform the parameter input, solution and the output (e.g. LaModel); however, the full-featured LaM2D program incorporates all of the numerical modeling steps into a single program.

Furthermore, the many of the material property input parameters necessary for modeling often appear daunting for practicing engineers who do not have the time to gather and verify specific material properties for both the seam and its surrounding rocks. In the LaM2D program, realistic average coal seam and overburden parameters are automatically defined. In fact, if the program is run in the standard mode, the only parameters required from the user are: the general model information which includes model title, number of seams, and the system of units to be used; the seam geometry information which includes element size, grid size, seam depths, and thicknesses; and the seam material grids. In standard mode, all of the seam and overburden properties are automatically entered with average values. However, if the user desires and the program is run in advanced mode, the program allows the user the total flexibility of adjusting any and all parameters in the program. For the seam material codes, an Excel-like grid editor is used to input the material codes into the program. The grid editor displays a cross-section of each of the modeled mining sections, and the user enters the coal and gob letter codes into the appropriate grid locations to represent the 2D mine layout using a mouse-driven spreadsheet-type interface.

Once all of the input has been completed, the LaM2D program typically runs in just a few seconds. Then, the output can be viewed using a number of plotting routines. The values output from the program include: seam convergence, seam stress, multiple-seam stress, multiple-seam subsidence, subsidence-based horizontal strain, and pillar safety factors. Any of these values can be plotted separately using the traditional “colored-square” plot or line graph. Also, a new comparative plot can be used to visualize, compare and analyze the interacting effect of two different output values from either seam.

5. CASE STUDY

In order to test the accuracy and utility of using the new simplified multiple-seam program, LaM2D, a comparative case study was performed using both LaModel and LaM2D. This case study was taken from the literature [4] and documents a situation where a longwall panel undermined an active room-and-pillar main line development. The upper mine
operates in the 9 ft thick Coalburg seam and has driven a 7 entry mainline with 12.2 by 24.4 m (40 by 80 ft) pillars. The lower mine operates in the 1.7 m (5.6 ft) thick No. 2 Gas and was retreating a 304.8 m (1000 ft) wide longwall panel directly under the overlying mains. The interburden at this site averaged 170.7 m (560 ft) and the maximum overburden over the upper mine was about 121.9 m (400 ft). The idealized mine map for this site as used in the comparative study is shown in Figure 4.

![Figure 4. Idealized mine plan for the case study analysis.](image)

(For the idealized comparative study in this paper, the mains are modeled as perfectly aligned with the middle of the longwall panel and all of the pillars in the mains and the gateroads are modeled with consistent sizes. In reality, the mains were somewhat offset from the center of the longwall panel and slightly skewed to the longwall advance. Also, the pillars in both the mains and the longwall gateroads have some variations in dimensions due to practical and operational considerations [4].

In the field, when the upper seam was under mined by the longwall, tension cracks began to develop in the roof when the underlying face approached to within 21.3 m (70 ft). Roof and rib control problems intensified as the longwall moved under and continued beyond the upper seam. The sandstone roof in the upper mine was extensively fractured with some apertures measured at 101.6 mm (4 in). Several roof falls occurred and severe rib spalling was experienced. As the dynamic subsidence settled and reached equilibrium, the roof fractures mostly closed and conditions significantly improved.

For the mine plan in Figure 4, a 3 dimensional LaModel model was developed. This model used 10 ft elements on a 250 X 250 grid to represent the mine plan shown. To enter the material parameters and the mine grid in order build this simplified model consumed about one hour of time (for the real mine plan with variable size pillars and off-angle pillar, more that a day was used to build the model). The program then took another hour to solve the model for the seam stresses and displacements. Overall, LaModel took about 2 hours to build and solve the model.

Using LaM2D, a 2 dimensional cross-section from the critical area (section A-A’) in Figure 4 was modeled. This cross-section essentially consists of the upper seam pillars and the lower seam longwall as pictured in Figure 5. With the simplified, 2 dimensional LaM2D, a linear array of 250 – 3 m (10 ft) elements was used to represent each seam. The minimum parameter input for the model was used. After all of the parameters and grids were entered, the program solved for the seam stresses, displacements, safety factors and strains in a few seconds. Overall, the total parameter input and solution required less than 10 minutes for the LaM2D.

![Figure 5. Cross section used in the LaM2D program.](image)

Figure 6 shows a comparison between the stresses calculated with the 3D LaModel program and those calculated with the simplified LaM2D program. For all practical purposes, the vertical stresses calculated by the two models match reasonably well. Both
models show the stress abutment in front of the advancing longwall face and the reduced stress in the gob behind the longwall. The unusual behavior at the east side of the simulated cross section was as a result of the model edge effects. Further, the LaM2D program can be used to determine the pillar safety factors and the subsidence induced strain. For instance, the pillar safety factors for the case study are shown in Figure 7.

![Figure 6. Comparison of LaModel and LaM2D stresses.](image)

![Figure 7. LaM2D safety factor output.](image)

It can be seen in this figure that the safety factors are generally high and pillar failure would not be expected. This is consistent with observations in the field which reported pillar spalling but no pillar failure.

The vertical subsidence and horizontal strain was also calculated for the case study as shown in Figures 8 and 9. The calculated subsidence in the upper seam is essentially 1.07 m (3.5 ft) as reported in the case study literature [4]. This subsidence generated tensile and compressive strains in conjunction with the moving longwall on the order of 2,300 microstrain. Based on the National Coal Board experience with surface strains [18], the 2,300 microstrain would cause “medium to severe damage”. This level of damage appears to correlate well with the observed roof cracking and roof falls.

![Figure 8. Case study subsidence](image)
6. SUMMARY AND CONCLUSIONS

A simplified, two-dimensional boundary-element method designed to model the complex multiple-seam stress and displacement interactions has been implemented into the LaM2D program. This new program allows the user to quickly and easily calculating the stresses, displacements, safety factors and strains associated with basic multiple-seam mining situations that can be accurately represented in two dimensions. The input to LaM2D has been greatly simplified by incorporating automatic overburden, interburden, coal and gob property generation. Also, the program is the first multiple-seam analysis package to inherently calculate pillar safety factors and multiple-seam induced subsidence and strain. In a practical comparison with the full-featured multiple-seam program, LaModel, the simplified LaM2D provided comparable stress and displacement calculations in minutes instead of hours. Further, the unique safety factor and strain calculations of the LaM2D program provided practical accurate assessments of the observed pillar and roof behavior in the modeled mine.

REFERENCES


